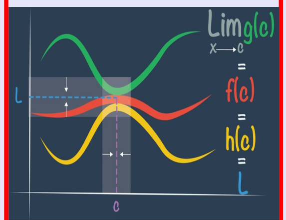
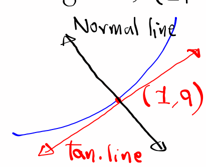
Math 261

Fall 2022

Lecture 19



Find equation of tangent line and normal line to the curve given by $f(x) = (1+2x)^2$ at $(1, 9)$ ✓



Verify the Point

$f(1) = (1 + 2\cdot 1)^2 = 3^2 = 9$ ✓

Find $f'(x)$

$f(x) = (1+2x)^2 = (1+2x)(1+2x)$

Product Rule

$f'(x) = 2(1+2x) + (1+2x)\cdot 2$

$= 4(1+2x)$

$m_{\text{tan. line}} = f'(1) = 4(1+2\cdot 1) = 12$

Eqn. of tan. line

$y - y_1 = m(x - x_1)$

$y - 9 = 12(x-1)$

$\boxed{y = 12x - 3}$

$m_{\text{Normal line}} = \dfrac{-1}{m_{\text{tan. line}}} = \dfrac{-1}{12}$

Equation of Normal line

$y - y_1 = m(x - x_1)$

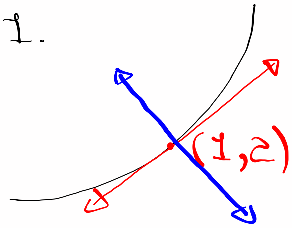
$y - 9 = \frac{-1}{12}(x-1)$

$\to y = \frac{-1}{12}x + \frac{1}{12} + 9$

$\boxed{y = \frac{-1}{12}x + \frac{109}{12}}$

Find eqn of the normal line to the curve given by $f(x) = \dfrac{3x+1}{x^2+1}$ at the point with $x = 1$.

$f(1) = \dfrac{3(1)+1}{1^2+1} = \dfrac{4}{2} = \boxed{2}$



$m_{\text{Normal line}} = \dfrac{-1}{f'(1)}$

$f(x) = \dfrac{3x+1}{x^2+1}$ $\qquad f'(x) = \dfrac{3(x^2+1) - (3x+1)\cdot 2x}{(x^2+1)^2} = \dfrac{df}{dx}$

$f'(1) = \dfrac{3\cdot 2 - 4\cdot 2}{(1+1)^2} = \dfrac{-2}{4} = \boxed{\dfrac{-1}{2}}$

$m_{\text{Normal line}} = \dfrac{-1}{f'(1)} = \dfrac{-1}{\frac{-1}{2}} = \boxed{2}$

$y - y_1 = m(x - x_1)$ $\qquad y - 2 = 2(x-1)$

$\boxed{y = 2x}$

Given $g(t) = \dfrac{2}{\sqrt[4]{t^3}}$ Find $\dfrac{dg}{dt}$

$g(t) = \dfrac{2}{t^{3/4}}$ $\qquad g(t) = 2\,t^{-3/4}$

$\dfrac{dg}{dt} = g'(t) = 2\cdot \dfrac{-3}{4}\, t^{-3/4 - 1} = \dfrac{-3}{2}\, t^{-7/4}$

$= \dfrac{-3}{2t^{7/4}} = \dfrac{-3}{2\sqrt[4]{t^7}} = \dfrac{-3}{2\sqrt[4]{t^4}\sqrt[4]{t^3}}$

$= \dfrac{-3}{2t\sqrt[4]{t^3}}$

given $f(x) = x^4 - 4x^3 + 16x$

find the first & second derivatives

$f'(x)$ $\quad$ $f''(x)$ $\quad$ f-double prime

$\frac{df}{dx}$ $\quad$ $\frac{d^2f}{dx^2} = \frac{d}{dx}\left[\frac{df}{dx}\right]$

$y =$ $\quad f(x) = x^4 - 4x^3 + 16x$

$y' =$ $\quad f'(x) = 4x^3 - 12x^2 + 16$

$y'' =$ $\quad f''(x) = 4 \cdot 3x^2 - 12 \cdot 2x + 0$

$$\boxed{f''(x) = 12x^2 - 24x}$$

Given $y = 2 + 3x^2 - x^3$

find y', and y''.

$y' = 0 + 6x - 3x^2$

$y'' = 6 - 6x$

Solve $y' = 0$ & $y'' = 0$

$y' = 0 \rightarrow 6x - 3x^2 = 0$ $\quad 3x(2-x) = 0$

$x = 0$ $\quad 2 - x = 0$

$\boxed{x=0}$ $\quad \boxed{x=2}$

$y'' = 0 \rightarrow 6 - 6x = 0$

$\boxed{x=1}$

Complete this chart

x		0		1		2	
y'	−	•	+		+	•	−
y''	+		+	•	−		−

Given $f(x)=\frac{x^2}{2x+1}$, find $\frac{df}{dx}$, $\frac{d^2f}{dx^2}$

$f'(x)$ $\quad f''(x)$

$$f'(x)=\frac{2x\cdot(2x+1)-x^2\cdot 2}{(2x+1)^2}=\frac{4x^2+2x-2x^2}{(2x+1)^2}=\frac{2x^2+2x}{(2x+1)^2}$$

$$f''(x)=\frac{\frac{d}{dx}[2x^2+2x]\cdot(2x+1)^2-(2x^2+2x)\cdot\frac{d}{dx}\left[(2x+1)^2\right]}{\left[(2x+1)^2\right]^2}$$

$$=\frac{(4x+2)(2x+1)^2-(2x^2+2x)\cdot 4(2x+1)}{(2x+1)^4}$$

$$\frac{d}{dx}\left[(2x+1)^2\right]=$$

$$\frac{d}{dx}\left[(2x+1)(2x+1)\right]=$$

$$2(2x+1)+(2x+1)\cdot 2=$$

$$4(2x+1)$$

$$=\frac{\cancel{(2x+1)}\left[(4x+2)(2x+1)-4(2x^2+2x)\right]}{(2x+1)^{\cancel{4}\,3}}$$

$$=\frac{\cancel{8x^2}\cancel{+4x}\cancel{+4x}+2\cancel{-8x^2}\cancel{-8x}}{(2x+1)^3}$$

$$\boxed{f''(x)=\frac{2}{(2x+1)^3}}$$

Given $f(x)=\frac{x}{x+\frac{c}{x}}$ Find $f'(x)$, and $f''(x)$

LCD $=x$

$$f(x)=\frac{x^2}{x^2+c}$$

$$f'(x)=\frac{2x(x^2+c)-x^2\cdot 2x}{(x^2+c)^2}=\frac{2cx}{(x^2+c)^2}$$

$$f''(x)=\frac{2c(x^2+c)^2-2cx\cdot 4x(x^2+c)}{\left[(x^2+c)^2\right]^2}$$

$$\frac{d}{dx}\left[(x^2+c)^2\right]=$$

$$\frac{d}{dx}\left[(x^2+c)(x^2+c)\right]=$$

$$2x(x^2+c)+(x^2+c)\cdot 2x=$$

$$4x(x^2+c)$$

$$=\frac{2c\cancel{(x^2+c)}\left[x^2+c-4x^2\right]}{(x^2+c)^{\cancel{4}\,3}}$$

$$=\boxed{\frac{2c(c-3x^2)}{(x^2+c)^3}}$$

Find a quadratic function such that

$f(2)=5$, $f'(2)=3$, and $f''(2)=2$.

$f(x)=ax^2+bx+c$

$f'(x)=2ax+b$

$f''(x)=2a$

$f''(2)=2$

$2a=2 \rightarrow \boxed{a=1}$

$\boxed{b=-1}$

$\boxed{c=3}$

$\boxed{f(x)=x^2-x+3}$

$f(2)=5$

$a(2)^2+b(2)+c=5$

$\boxed{4a+2b+c=5}$

$f'(2)=3$

$2a(2)+b=3$

$\boxed{4a+b=3}$

$4(1)+b=3$

$4a+2b+c=5$

$4(1)+2(-1)+c=5$

Find a parabola that has slope 4 at $x=1$, slope -8 at $x=-1$ and it contains $(2,15)$

$y=ax^2+bx+c$

$y'|_{x=1}=4$

$y'|_{x=-1}=-8$

$y'=2ax+b$

$a(2)^2+b(2)+c=15$

$\boxed{4a+2b+c=15}$

$2a(1)+b=4$

$\boxed{2a+b=4}$

$2a(-1)+b=-8$

$\boxed{-2a+b=-8}$

$$\begin{cases} 4a+2b+c=15 \\ 2a+b=4 \\ -2a+b=-8 \end{cases}$$

$2b=-4$

$\boxed{b=-2}$

$2a+b=4$

$2a-2=4$

$\boxed{a=3}$

$4a+2b+c=15$

$4(3)+2(-2)+c=15$

$8+c=15$

$\boxed{c=7}$

Parabola

$\boxed{y=3x^2-2x+7}$